THE ESTIMATION OF TRIPLE JUNCTION ANGLES IN OPAQUE SECTIONS OF ANNEALED METALS AND METAMORPHIC ROCKS

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(with 5 Text-figures and 2 Tables)

ABSTRACT

The statistical procedure advocated by Harker & Parker (1945) for the estimation of true triple junction angles in annealed aggregates of opaque material, is questioned in the light of more recent data from transparent sections suggesting that the true angle may itself have a natural variation even at equilibrium. A computer simulation technique is used to show that the mean of the true triple junction angle can be estimated (with varying precision) from the frequency distribution of apparent angles provided that the natural variation of the true angle does not have a standard deviation greater than about 20°. Theoretically, the standard deviation of the true angle variation can also be deduced from the apparent angle distribution but it is shown that this is only feasible if the standard deviation of the true triple junction angle is greater than 10°.

Published apparent angle distributions do not conform to the theoretical distribution and it is suggested that they are similar to those that would be obtained if triple junctions making low angles with the section plane are not measured. This restricts the information that can be obtained and questions the validity of some results.

INTRODUCTION

When a granular aggregate such as a metal ceramic or rock is deformed and then annealed at an elevated temperature, the individual grains readjust to form shapes that are dictated by the requirements of space filling and the minimizing of interfacial free energies (Smith, 1948). The equilibrium configuration in an isotropic aggregate is an even grained array of polygonal grains with planar or smoothly curved interfaces tending to meet three at a time in a line known as a triple junction (Fig. 1a).

The angle subtended by any pair of the three interfaces meeting at a triple junction is a function of the specific surface free energies of the interfaces which, in the absence of surface impurities, are numerically equivalent to the interfacial tensions (Kretz, 1966). Thus, the junction can be represented in section by interfacial tension vectors (Fig. 1b). From Fig. 1b (after Smith, 1948; Kretz, 1966, etc.), it can be seen that the interfacial tensions (γ) and the included angles (θ) are related by:

$$\frac{\gamma_{23}}{\sin\theta_1} = \frac{\gamma_{13}}{\sin\theta_2} = \frac{\gamma_{12}}{\sin\theta_3} \qquad (1)$$

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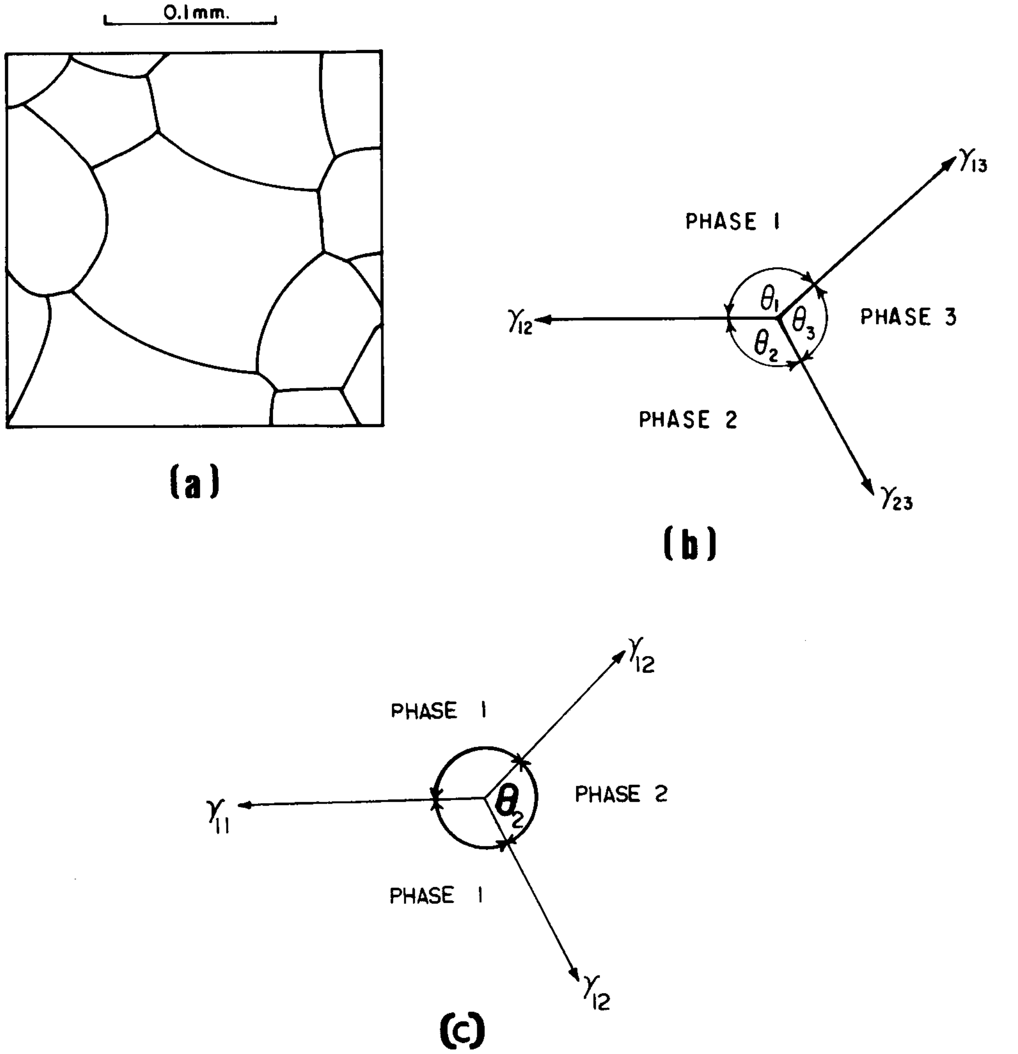

Fig. 1 (a) A random section through triple junctions in annealed iron (*after* Smith, 1948).

(b) Interfacial tension vectors (γ) at a triple junction in a three phase aggregate. The γ_{12}, etc. are the specific interfacial tensions between phases 1 and 2, etc.

(c) Interfacial tension vectors (γ) at a two phase triple junction. θ_2 is the dihedral angle.

If the aggregate is a single phase then $\gamma_{12} = \gamma_{13} = \gamma_{23}$ and hence

$$\theta_1 = \theta_2 = \theta_3 = 120° \text{ at equilibrium.}$$

Similarly, in a two phase aggregate at equilibrium (Fig. 1c)

$$\gamma_{11} = \gamma_{12} \cos(\theta_2/2)$$

where θ_2, the dihedral angle, can have any characteristic value.

The microstructure that is developed in an annealed aggregate is a function of the relative interfacial free energies and the proximity to equilibrium. The measurement of triple junction angles is one method used to evaluate both these parameters and to interpret the microstructure. It is a common procedure in metallography (Chadwick, 1972) and to some extent has been applied to rocks. For example, Stanton (1964), Kretz (1966), Vernon (1968), Spry (1969), etc. have interpreted the metamorphic history of rocks from the microstructure on the basis of measured triple junction angles (assuming equilibrium), while Stanton & Gorman (1968) have determined the proximity to equilibrium using triple junction angle measurements. Qualitative evaluation of the triple junction relationships are implicit in many studies (e.g. Rast, 1965).

MEASUREMENT and STATISTICAL PROBLEMS

Triple junction angles in thin sections of transparent minerals present no real measurement problems. Each junction can be so oriented on a universal stage that the true triple junction angle can be measured. (The angle measured in a plane normal to the junction line is the true angle). In sections involving opaque minerals this method is not normally feasible and only apparent angles can be measured.

The remainder of this discussion is concerned with the problems of estimating the true triple junction angle in opaque sections. A random section through an aggregate of polygonal grains can cut any given triple junction angle such that the observed apparent angle can be anything between 0° and 180°, although with greatly differing probabilities. Harker & Parker (1945) showed that if all the randomly oriented triple junction angles (or dihedral angles) have the same true value in the aggregate then the most probable apparent angle will also be the true value. The probability distribution of apparent angles for true dihedral angles of 90° and 120° are shown in Fig. 2a.

The procedure suggested by Harker & Parker (1945) and elaborated by Smith (1948) is to measure a large number of apparent dihedral angles and to plot their frequency distribution. If the distribution agrees closely with the theoretical distribution, then that angle most frequently observed will be the true dihedral angle. In reported practice, however, it appears that the step comparing the observed with the theoretical distribution has usually been ignored. One aim of this paper is to examine the validity of that practice.

If one examines published distributions of apparent triple junction angles in metals or sulphides (e.g. Fig. 2b) then two anomalies are apparent. The frequency of the modal class is commonly 10% to 15% lower than the theoretical frequency and the dispersion is much less than it theoretically should be. This latter is reflected in ranges that rarely exceed 120° (and commonly are less than 100°) and lower standard deviations than expected.

There are several reasons why an observed apparent angle distribution may be different from the theoretical distribution, even if their modes are the same:

1) It may be that the grain shape has not yet reached equilibrium and one would expect that the distribution would "peak up" as equilibrium is approached. Stanton & Gorman (1968) use the change in standard deviation of the apparent angle distribution in this way as a measure of the rate of annealing.
2) Even at equilibrium the true dihedral angle may deviate significantly about a mean value. Kretz (1966) and Vernon (1968) show that this is due to the variation of interfacial free energy with orientation and that the true angle may itself have a distribution with a standard deviation up to 20°, depending on the mineral.
3) Triple junction lines that are oriented non-randomly also may be a factor. Since the statistical procedure of Harker & Parker (1945) is not applicable to aggregates with a preferred orientation, this factor will not be considered in this discussion.

In either of cases (1) or (2) above, the observed distribution of apparent dihedral angles is a combination of two separate distributions, one being the natural variation of the true angle and the other being the probability distribution for the true angle with no variation. Theoretically, it should be possible to extract the standard deviation of the natural variation from the observed apparent angle distribution. It can be shown from basic statistics (e.g. Freund, 1972, p.195, Theorem 6.2) that:

$$\sigma_T^2 = \sigma_0^2 - \sigma_A^2 \qquad (2)$$

where: σ_T = standard deviation of the natural variation in the true angle.
σ_0 = standard deviation of the observed apparent angle distribution.
σ_A = standard deviation of the apparent angle distribution with no natural variation.

(In this paper, the symbol σ will refer to the population standard deviation, and the symbol S will refer to the sample standard deviation.)

No attempt to derive this natural variation in the true angle appears to have been attempted in published data although, for example, S_T may have been a more appropriate measure of the progress of annealing in Stanton and Gorman's (1968) experiments. The problem lies in the fact that theoretically σ_0 should be greater than either σ_T or σ_A and, as discussed above, this is rarely the case in recorded measurements. Stanton & Gorman (1968) quote S_0 values for fully annealed galena and sphalerite in the range from 9° to 13° whereas the σ_A value derived from Harker & Parker (1945) is 22.12°.

In summary, the problems associated with the determination of dihedral

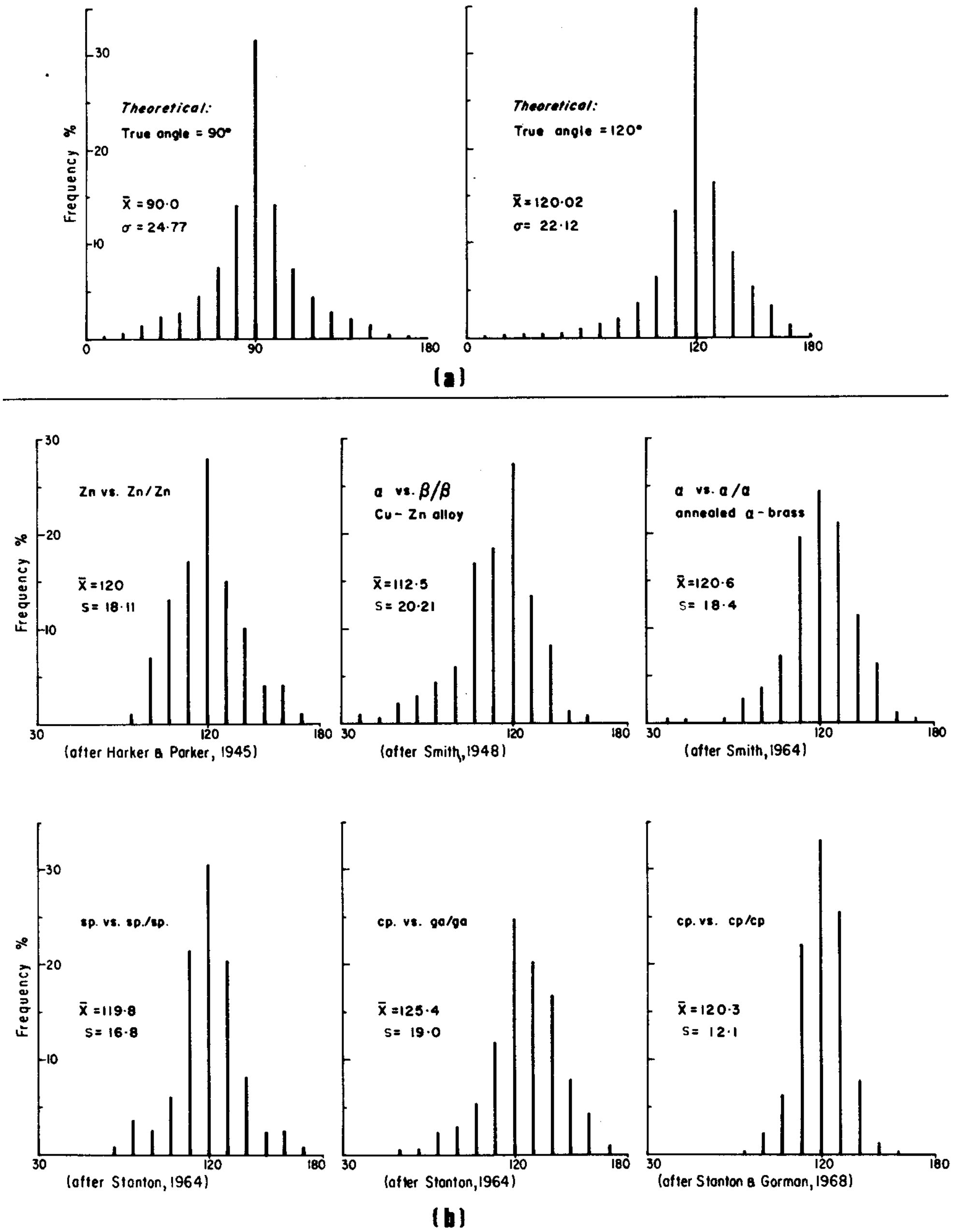

Fig. 2 (a) Theoretical frequency distributions of apparent triple junction angles for true angles of 90° and 120° (*after* Harker & Parker, 1945).

(b) Observed apparent triple junction angle distributions in various annealed aggregates (sp = sphalerite; ga = galena; cp = chalcopyrite).

angles at triple junctions in opaque specimens pose the following questions:

(1) What is the effect on theoretical apparent angle distributions such as those shown in Fig. 2a if the true angle also has a natural variation with standard deviation σ_T?
(2) Is the mean or the mode of the observed distribution the better measure of the true triple junction angle?
(3) How large can σ_T be before the true angle cannot be estimated with any great confidence? That is to say, to what precision can the true angle be estimated for any given σ_T?
(4) Can S_T always be extracted from the S_0 of the observed distribution?
(5) Why do most reported apparent angle distributions have a standard deviation (S_0) much lower than expected?

COMPUTER SIMULATION OF APPARENT DIHEDRAL ANGLE DISTRIBUTIONS

A computer simulation of the measurement of apparent dihedral angles has been established in order to evaluate the various statistical problems outlined. Random numbers are used to define the position of a randomly oriented triple junction with respect to a plane of section (see Appendix). The apparent angle produced can be calculated and the process reiterated any number of times to simulate separate readings. Simultaneously other random numbers are used to describe the true angle which has a normal distribution about any given mean and with any given standard deviation, σ_T.

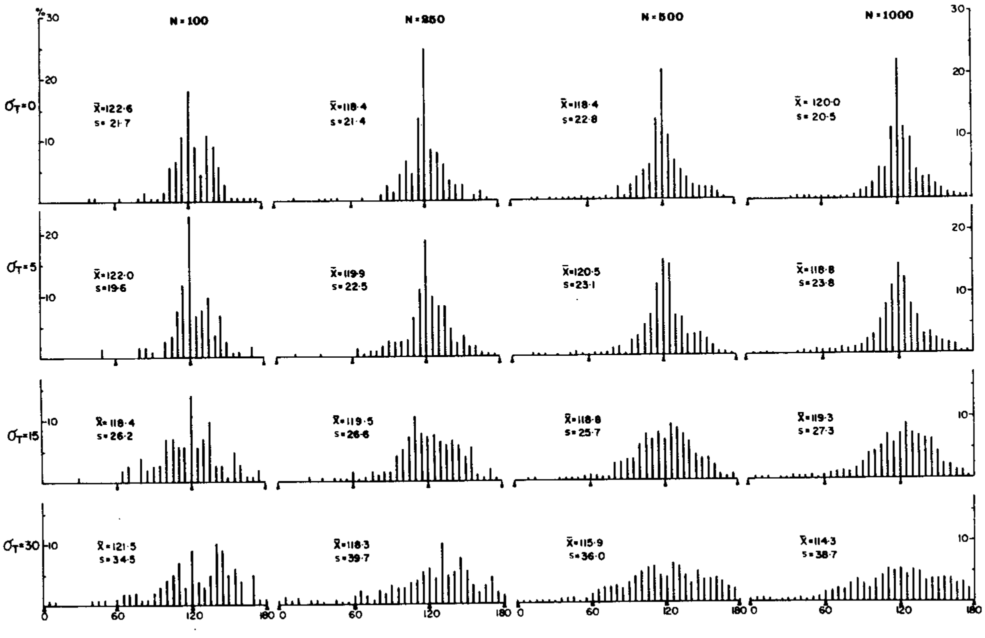

Fig. 3 Computer simulated distributions of apparent triple junction angles on random sections. σ_T = standard deviation of the true angle about 120° mean. N = number of "readings". $\bar{x}$ = mean of observed distribution. S = standard deviation of observed distribution.

Some results of this simulation are shown in Tables 1 and 2 and in Fig. 3 for the case in which the true dihedral angle is 120°. Tables 1 and 2 describe in detail the statistics of simulated apparent angle distributions for various true angle standard deviations (σ_T) and various numbers of readings (N). σ_T ranges from 0° to 30° and N varies from 50 to 1000 "readings". The statistics from two separate runs are shown together and all statistics are given to a 95% confidence level. Bar graphs of some of the distributions given in the Tables are shown in Fig. 3. Graphs from each row have the same standard deviation of the true angle (σ_T) and graphs in each column represent the same number of "readings" (N). The sample mean ($\bar{x}$) and standard deviation (S) of each distribution is also shown. Bar graphs of this type are the forms of presentation normally used.

In Table 2(c) equation (2) is used to estimate the standard deviation of the true angle, S_T, from the observed distribution. Where "–" is shown, the standard deviation of the observed distribution, S_0, is less than that of the theoretical distribution, σ_A. (For a true angle of 90°, σ_A is 24.77°; for a true angle of 120°, σ_A is 22.15°). The precision of this estimation, shown in Table 2(d) is derived from a calculation of the estimated true angle using S_0 plus or minus the precision interval shown in Table 2(b).

DISCUSSION

Since the work of Harker & Parker (1945), general practice has been to take the most common value of the apparent angle distribution (i.e. the mode or modal class) as an estimate of the true dihedral angle. From Table 1 it is apparent that the mean is a more stable statistic and a better estimation of the true angle than the mode. For $\sigma_T > 1°$ the mode becomes unreliable although the modal class of 5° interval is reliable for σ_T up to 10°. However, the positioning of the modal class is generally dependent on the mode. A more serious drawback to the use of the mode is its greater unreliability for low numbers of readings than the mean. An added advantage of the mean is that its precision can be estimated as shown in Table 1(b).

As σ_T becomes large (e.g. $\sigma_T \geqslant 30°$) the mean of the apparent angle distribution shown in Table 1(a) becomes somewhat lower than the mean of the true angle. This is due to the skewness of the apparent angle distribution for a true angle of 120° (Fig. 2a). Thus for $\sigma_T \geqslant 30°$ the mean of the true dihedral angle can only be estimated with very low precision. This problem does not arise for a true angle of 90° as the apparent angle distribution is symmetrical.

Even though the mean of the true angle distribution cannot be estimated easily as σ_T becomes large, the estimation of S_T from S_0 using equation (2) is quite valid and reliable. However, as the true σ_T drops below about 10° this estimation becomes quite unreliable (Table 2[c]). The reason can be seen from the theoretical curve of σ_0 (Fig. 4). Below a σ_T of 10° the σ_0 values rapidly approach a constant value (σ_A). Thus, the estimation of σ_T becomes swamped in the imprecision of determining σ_0.

As discussed, most reported values of S_0 from observed apparent angle

TABLE 1

Mean

N = 50	100	250	500	750	1000
120.6	122.6	118.4	118.4	117.9	120.0
120.1	119.8	118.0	119.9	118.8	119.8
119.8	118.4	118.6	117.5	119.7	119.9
118.6	120.9	119.2	120.0	119.5	119.9
118.2	122.0	119.9	120.5	119.0	118.8
123.7	119.7	120.3	119.8	119.2	120.7
120.2	121.1	118.4	120.6	120.3	119.4
116.2	118.3	119.8	119.4	118.5	119.7
112.2	118.4	119.5	118.8	117.7	119.3
118.1	119.3	119.7	121.2	120.7	120.3
119.8	121.5	118.3	115.9	115.8	114.3
118.3	118.8	111.2	117.2	115.0	116.5

(a)

Precision of Mean (95% confidence)

σ_T	N = 50	100	250	500	750	1000
	6.1	4.3	2.7	2.0	1.6	1.3
0%	5.8	4.0	2.8	2.1	1.6	1.4
	7.0	4.4	2.8	2.1	1.6	1.3
1%	7.4	3.9	3.0	1.9	1.5	1.4
	4.8	3.8	2.8	2.0	1.7	1.5
5%	6.1	4.9	2.5	2.0	1.7	1.3
	6.1	4.5	3.0	1.9	1.6	1.5
10%	8.2	4.9	3.0	2.1	1.7	1.5
	7.7	5.1	3.3	2.3	1.9	1.7
15%	6.6	5.8	3.3	2.3	1.8	1.6
	9.7	6.8	4.9	3.2	2.6	2.4
30%	10.8	7.4	4.7	3.3	2.8	2.4

(b)

Mode

N = 50	100	250	500	750	1000
123	119	119.5	119	120	120
120	119	119	121	120	119
bim.	119	118	120	120	120
119.5	120	121	120	119	120
poly.	119	121	116	120	122
bim.	121	123	126	bim.	125
poly.	125	120.5	126	122	122
138	131	123	128	125	119
poly.	poly.	112	104	109	bim.
poly.	133	122	111	117	132
poly.	poly.	132	bim.	113	122
130	poly.	poly.	bim.	140	124

(c)

Modal Class (5° interval)

σ_T	N = 50	100	250	500	750	1000
	125	120	120	120	120	120
0%	120	120	120	120	120	120
	120	120	120	120	120	120
1%	120	120	120	120	120	120
	125	120	120	120	120	120
5%	bim.	120	120	125	120	125
	125	125	bim.	125	120	115
10%	125	120	125	120	120	120
	105	120	110	125	135	125
15%	poly.	poly.	120	115	115	120
	135	140	130	125	115	120
30%	165	100	120	135	125	140

(d)

TABLE 1 Statistics of the *average* of computer generated apparent angle distributions

TABLE 1

Mean							Precision of Mean (95% confidence)					
N = 50	100	250	500	750	1000	σ_T	N = 50	100	250	500	750	1000
120.6	122.6	118.4	118.4	117.9	120.0		6.1	4.3	2.7	2.0	1.6	1.3
120.1	119.8	118.0	119.9	118.8	119.8	*0%*	5.8	4.0	2.8	2.1	1.6	1.4
119.8	118.4	118.6	117.5	119.7	119.9		7.0	4.4	2.8	2.1	1.6	1.3
118.6	120.9	119.2	120.0	119.5	119.9	*1%*	7.4	3.9	3.0	1.9	1.5	1.4
118.2	122.0	119.9	120.5	119.0	118.8		4.8	3.8	2.8	2.0	1.7	1.5
123.7	119.7	120.3	119.8	119.2	120.7	*5%*	6.1	4.9	2.5	2.0	1.7	1.3
120.2	121.1	118.4	120.6	120.3	119.4		6.1	4.5	3.0	1.9	1.6	1.5
116.2	118.3	119.8	119.4	118.5	119.7	*10%*	8.2	4.9	3.0	2.1	1.7	1.5
112.2	118.4	119.5	118.8	117.7	119.3		7.7	5.1	3.3	2.3	1.9	1.7
118.1	119.3	119.7	121.2	120.7	120.3	*15%*	6.6	5.8	3.3	2.3	1.8	1.6
119.8	121.5	118.3	115.9	115.8	114.3		9.7	6.8	4.9	3.2	2.6	2.4
118.3	118.8	111.2	117.2	115.0	116.5	*30%*	10.8	7.4	4.7	3.3	2.8	2.4

(a) (b)

Mode							Modal Class (5° interval)					
N = 50	100	250	500	750	1000	σ_T	N = 50	100	250	500	750	1000
123	119	119.5	119	120	120		125	120	120	120	120	120
120	119	119	121	120	119	*0%*	120	120	120	120	120	120
bim.	119	118	120	120	120		120	120	120	120	120	120
119.5	120	121	120	119	120	*1%*	120	120	120	120	120	120
poly.	119	121	116	120	122		125	120	120	120	120	120
bim.	121	123	126	bim.	125	*5%*	bim.	120	120	125	120	125
poly.	125	120.5	126	122	122		125	125	bim.	125	120	115
138	131	123	128	125	119	*10%*	125	120	125	120	120	120
poly.	poly.	112	104	109	bim.		105	120	110	125	135	125
poly.	133	122	111	117	132	*15%*	poly.	poly.	120	115	115	120
poly.	poly.	132	bim.	113	122		135	140	130	125	115	120
130	poly.	poly.	bim.	140	124	*30%*	165	100	120	135	125	140

(c) (d)

TABLE 1 Statistics of the *average* of computer generated apparent angle distributions (see text for details).

TABLE 2

S_0

N = 50	100	250	500	750	1000
22.2	21.7	21.4	22.8	22.1	20.5
20.8	20.2	22.2	23.9	23.0	22.1
25.2	22.4	22.9	24.1	22.8	21.5
26.8	20.0	24.1	22.2	21.6	23.1
17.3	19.6	22.5	23.1	21.0	23.8
21.9	24.8	20.2	22.4	23.1	21.4
21.8	23.2	24.3	21.8	22.9	23.7
29.5	25.1	24.1	23.7	23.3	25.0
27.7	26.2	26.6	25.7	26.1	27.3
23.8	29.4	26.3	25.8	25.7	26.2
35.1	34.5	39.7	36.0	36.5	38.7
39.1	37.7	38.1	37.4	39.6	38.5

(a)

Precision of S_0
(95% confidence)

σ_T	N = 50	100	250	500	750	1000
	4.3	4.3	1.9	1.4	1.1	0.9
0%	4.1	2.8	2.0	1.5	1.2	1.0
	4.9	3.1	2.0	1.5	1.2	0.9
1%	5.3	2.8	2.1	1.4	1.1	1.0
	3.4	2.7	2.0	1.4	1.1	1.0
5%	4.3	3.4	1.8	1.4	1.2	0.9
	4.3	3.2	2.1	1.3	1.2	1.0
10%	5.8	3.5	2.1	1.5	1.2	1.1
	5.4	3.6	2.3	1.6	1.3	1.2
15%	4.7	4.1	2.3	1.6	1.3	1.1
	6.9	4.8	3.5	2.2	1.8	1.7
30%	7.7	5.2	3.3	2.3	2.0	1.7

(b)

Estimated S_T

N = 50	100	250	500	750	1000
1.3	-	-	5.6	-	-
-	-	2.4	9.2	6.2	1.0
12.0	3.4	6.1	9.4	5.5	-
15.2	-	9.5	1.4	-	6.5
-	-	4.3	6.6	-	-
-	11.2	-	3.8	6.7	-
-	6.9	10.1	-	6.1	8.4
19.6	11.9	9.7	8.6	7.4	11.7
16.6	14.1	14.7	13.2	13.8	16.1
8.8	19.4	14.2	13.4	13.1	14.0
27.2	26.4	33.0	28.4	29.1	31.8
32.2	30.5	31.0	30.2	32.9	31.5

(c)

Precision of Estimated S_T

σ_T	N = 50	100	250	500	750	1000
	13.3+	-	-	5.6+	-	-
0%	-	-	7.4+	5.3	6.2+	5.7+
	12.0+	9.2+	6.1+	5.0	5.5+	-
1%	15.2+	-	9.5+	6.7+	-	6.5+
	-	-	6.2+	6.6+	-	-
5%	-	-	11.2+	5.1+	6.7+	-
	-	7.5+	8.2	-	6.1+	3.6
10%	10.9	11.9+	9.7+	6.1	6.1	2.6
	14.2	9.5	4.8	3.5	2.7	2.1
15%	9.1+	7.0	4.9	3.4	2.8	2.3
	9.7	6.6	4.3	2.9	2.4	2.1
30%	9.9	6.8	4.2	2.9	2.4	2.1

(d)

TABLE 2 Statistics of the *dispersion* of the same computer generated distributions shown in Table 1.

distributions are much lower than expected. In this situation the previous discussion becomes meaningless, and it is difficult to interpret theoretically results such as those of Stanton & Gorman (1968), in which S_0 is used as a quantitative measure of the progress of annealing. (However, their general aim of qualitatively proving that annealing has occurred is amply demonstrated.) Although it is difficult to retrieve data from most published bar graphs, it appears that apparent angles less than 30° are very rare and, in the sulphide literature, reported angles of less than 60° are uncommon. For some reason, the small angles (which have a strong influence on the standard deviation) are not being measured. A possible reason is that low angles between a triple junction and the section plane may produce apparent angles that are very diffuse and difficult to either measure or possibly to even recognize as a triple point.

Fig. 5a shows the relationship between the orientation of a random section (see Appendix for the definition of the orientation parameters, θ and ϕ) and the observed apparent angle, A, for a true triple junction angle of 120°. θ is the angle between a triple junction and the normal to the section plane. It can be seen that if all the low junction angles are omitted (say, $\sin^2 \theta > 0.75$), then not only are all the extreme apparent angles not observed but even some of the angles around 120° are not observed.

Fig. 5b shows the results of computer simulations in which all the junctions inclined at less than 30° to the section ($\sin^2 \theta > 0.75$) are omitted. The apparent angle distributions produced are very similar to many of the published distributions. If the standard deviation of the true angle is very low, (0° say), the modal frequency is reduced and the standard deviation is of the order of 10° to 15°. The mean and the mode, however, still reflect the true triple junction angle.

CONCLUSIONS

Measurements of apparent angles in a random section through triple junctions can be used to estimate the true triple junction angle. The best estimate is the mean of the observed distribution rather than the mode. Even if the true angle has its own natural variation around some mean, this mean can be estimated with confidence if it is approximately 90°. If it is about 120° then it can still be estimated with reasonable confidence provided the natural variation does not have a standard deviation of 20° or more.

Provided that all precautions are taken to measure the entire apparent angle distribution (i.e. all junction orientations, even very low ones) then various statistics of the distribution can be used for quantitative purposes. In particular, it is possible to estimate the standard deviation of the true angle distribution provided that this is greater than about 10°.

Most published distributions from opaque aggregates do not show the entire apparent angle distribution and it is surmised that this is due to the omission of apparent angles of those triple junctions that are inclined at low angles to the section plane. Even in this case, it is found that the mean of the apparent angle distribution is still a good estimate of the true angle although

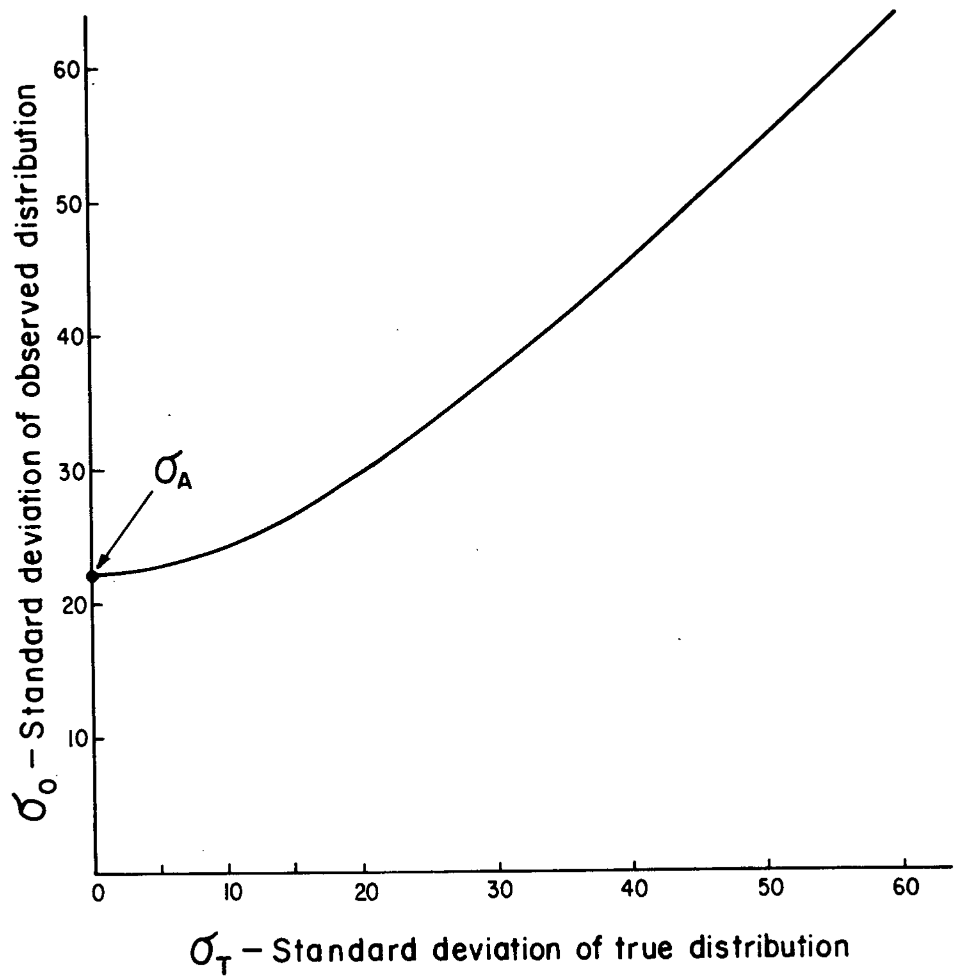

Fig. 4 Theoretical relationship between the standard deviation of the true angle variation (σ_T) and the standard deviation of the observed apparent angle distribution (σ_0) for a mean true angle of 120°. Note that for σ_T less than 10° there is little variation in σ_0.

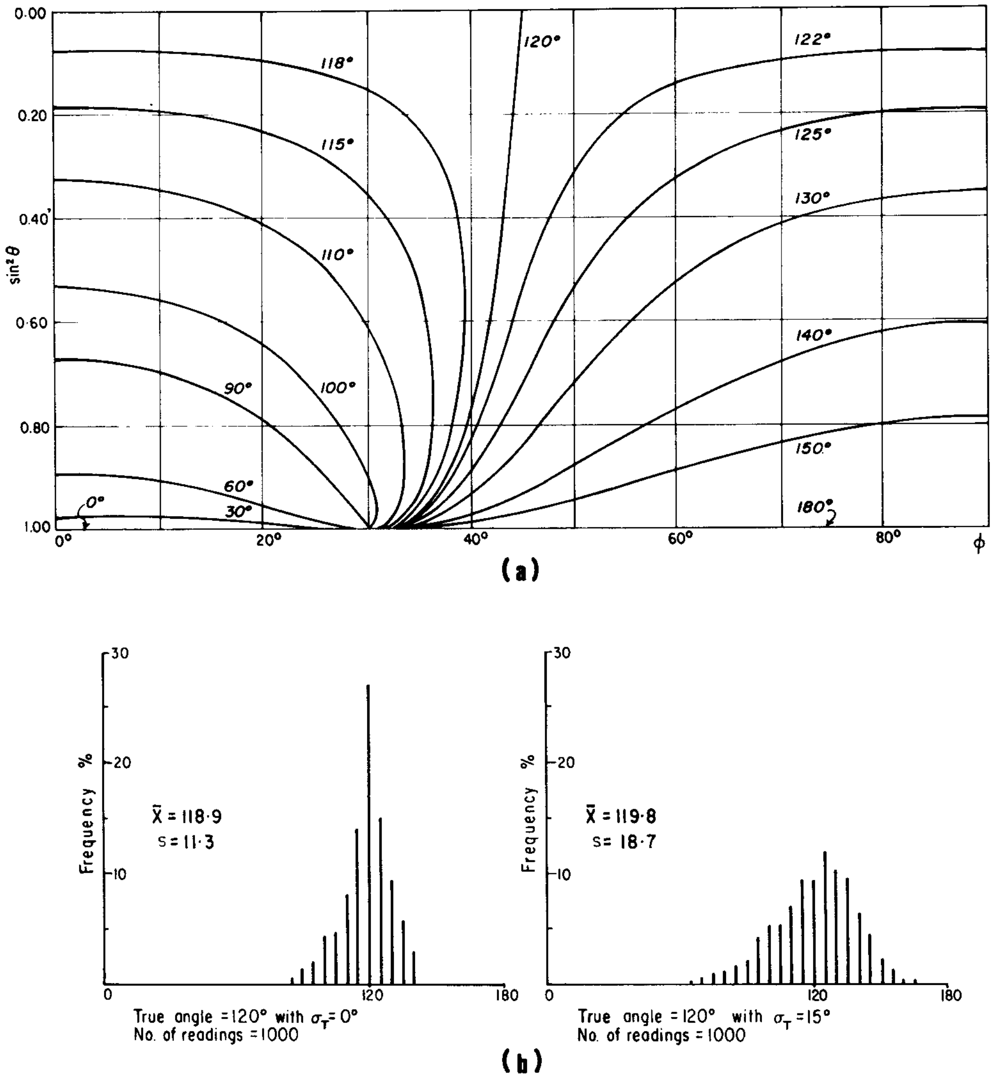

Fig. 5 (a) Contours of equal A on a map of $\sin^2\theta$ and ϕ for a true triple junction angle of 120°. Areas on this map are proportional to the probabilities of the corresponding ranges of A (*after* Harker & Parker, 1945).

(b) Computer simulated distributions of apparent triple junction angles if those junctions inclined at less than 30° to the section plane are omitted.

other statistics should not be used for quantitative purposes.

(In a recent publication that has come to the attention of this author since this paper was originally submitted, Lindh (1976) presents some apparent angle distributions from measurements on the dihedral angles of pyrrhotite, sphalerite and hematite at their triple junctions with quartz/quartz boundaries. He shows that none of the distributions fit the theoretical distributions very well (χ^2 – test, 5% significance) and suggests that this may be due to having overlooked those triple junctions that are at a low angle to the section.)

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APPENDIX

Method of Computer Simulation

The relationship between the true angle, T, at a triple junction, the apparent angle, A, and the position of a random section can be described by setting up a cartesian coordinate system related to the triple junction as follows: the z-axis is along the triple junction; the x-axis is normal to the z-axis and bisects the triple junction angle; and the y-axis is normal to both these (see Harker & Parker, 1945, Fig. 10).

A set of spherical polar coordinates can then be set up such that:

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

θ and ϕ can be considered as describing the colatitude and longitude, respectively, of the point of impingement of the normal to the section on a unit sphere. The apparent angle, A, can be described by the relationship:

$$\tan A = \frac{2 \sin T \cos\theta}{\sin^2\theta(\cos 2\phi - \cos T) + 2 \cos T} \quad \text{(Harker \& Parker, 1945)}$$

Harker & Parker (1945) showed that the probability of a triple junction making an angle θ with the normal to a random section is proportional to $\sin^2\theta$. Thus, random numbers between 0 and 1 and between 0 and 360 can be given to $\sin^2\theta$ and ϕ, respectively, to represent the probable orientation of random triple junctions cut by a random section plane. These values of $\sin^2\theta$ and ϕ can be substituted in the above equation to derive the apparent angle.

A Monte Carlo sampling method, suggested by Naylor *et al.* (1966) is used to generate random numbers with a normal distribution of a specified mean and standard deviation. The random number so chosen is then substituted for T in the above equation to simulate the natural variation in the true angle.