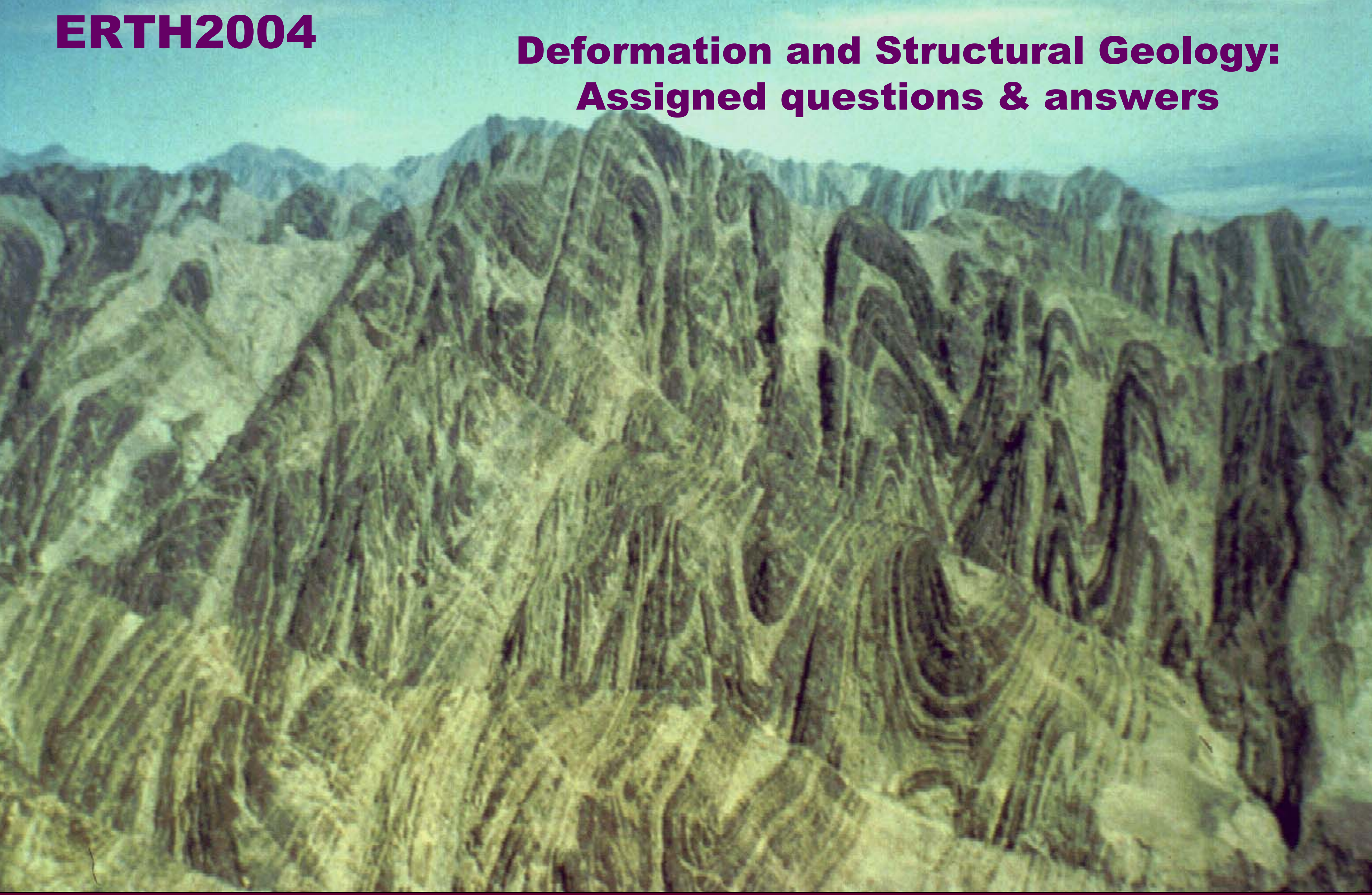
ERTH2004

Deformation and Structural Geology: Assigned questions & answers



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Strain Exercises

1. Strains in two areas have strain ratios respectively of X:Y:Z = 8:6:1 and 9:3:1. Which area is more deformed and what type of fabrics would you expect in each? [Hint: plot the strains on a Ramsay diagram].

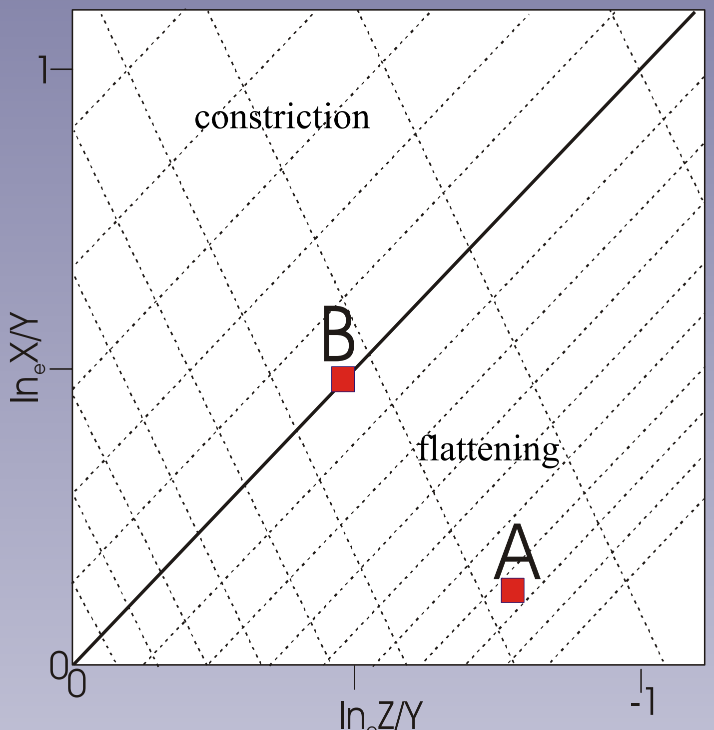
2. The ellipticity (X:Z) of a strain ellipse measured using deformed brachiopods on a horizontal bedding plane is 1.7:1. Approximately the same amount of strain is recorded in rocks across a broad foldbelt about 100km wide, with the principal shortening direction perpendicular to the trend of the fold belt. Estimate the amount of contraction, in kilometres, that has occurred across this 100 km-wide fold belt, assuming: a) that there has been no volume change; and b) that there has been a 15% volume decrease.

3. Ductile geological strain rates are of the order of 10^{-14} /sec. At that strain rate how long would the deformation event in the previous question have taken? (Note that because strain is dimensionless the units of strain rate are sec^{-1}).

Answers to Strain Exercises (1)

1. Strains in two areas have strain ratios respectively of X:Y:Z = 8:6:1 and 9:3:1. Which area is more deformed and what type of fabrics would you expect in each?

Method: Plot on Ramsay Plot



A. X:Y:Z = 8:6:1

$\therefore$ X/Y = 8/6 & Z/Y = 1/6

i.e. $\log_{10}$ X/Y = 0.12 & $\log_{10}$ Z/Y = 0.78

B. X:Y:Z = 9:3:1

$\therefore$ X/Y = 9/3 & Z/Y = 1/3

i.e. $\log_{10}$ X/Y = 0.48 & $\log_{10}$ Z/Y = -0.48

i.e A is more deformed than B (from plot)

A fabrics will be S-tectonite

B fabrics will be LS-tectonite

Answers to Strain Exercises (2a)

2. The ellipticity (X:Z) of a strain ellipse measured using deformed brachiopods on a horizontal bedding plane is 1.7:1. Approximately the same amount of strain is recorded in rocks across a broad fold belt about 100km wide, with the principal shortening direction perpendicular to the trend of the fold belt.

a) Estimate the amount of contraction, in kilometres, that has occurred across this 100km-wide fold belt, assuming: that there has been no volume change…

Method: Convert to stretches assuming no volume (area) change

$X : Z = 1.7 : 1$ i.e. $X = 1.7Z$ (i)

If no volume (area) change then: $X \cdot Z = 1$ (ii)

Substitute (i) in (ii): $1.7Z \times Z = 1$

i.e. $1.7 Z^2 = 1$

$\therefore Z = \sqrt[2]{(1/1.7)} = 0.77$

Substitute back in (i) gives: $X = 1.3$

$\therefore$ $X : Z = 1.3 : 0.77$ (expressed as Principal Stretches)

$\therefore$ if the final fold belt is 100 km wide (l_1) in the Z direction then its original width (l_0) is:

$l_1/l_0 = 100/l_0 = 0.77$ (iii)

i.e. $l_0 = 100/0.77 =$ **130 km (i.e. 30 km of contraction)**

Answers to Strain Exercises (2b)

....

b) that there has been a 15% volume decrease.

If the fold belt has lost 15% volume then Eq (ii) becomes:

X • Z = 0.85

FFollowing through the substitutions the Principal Stretches become:

X : Z = 1.4 : 0.71

Then Eq (iii) becomes: $100/l_0 = 0.71$

i.e. $l_0 = 100/0.71 =$ **141 km (i.e. 41 km of contraction)**

Answers to Strain Exercises (3)

3. Ductile geological strain rates are of the order of 10^{-14} /sec. At that strain rate how long would the deformation event in the previous question have taken? (Note that because strain is dimensionless the units of strain rate are sec^{-1})

Note that we cannot simply divide the finite strain measured as a stretch by the total time to get the instantaneous strain rate.

Instead the instantaneous strain rate is equal to the natural, or logarithmic, strain divided by the time: i.e. $\dot{\varepsilon} = \ln(l_1/l_0)/t$

The natural strain across the fold belt is:

$\varepsilon = \ln(l_1/l_0) = \ln(100/130) = \ln(0.77) = -0.26$

Now the question becomes:

If it takes 1 second to achieve a natural strain of 10^{-14th}, how long would it take to achieve a strain of 0.26?

…..

Answers to Strain Exercises (3) cont

If it takes 1 second to achieve a natural strain of 10^{-14th}, how long would it take to achieve a strain of 0.26?

i.e. Time = $(1 \times 0.26)/10^{-14}$ seconds

= $(0.26 \times 10^{14})/(60 \times 60 \times 24 \times 365 \times 10^{6})$ million years

= 0.82 my (820000 years)

Note that at these strain rates a typical mountain belt (orogen) with a bulk shortening of ~60% would take ~3my to form

– which, geologically, is a very short time

Natural ductile strain rates are probably a little slower than 10^{-14} but orogens of ~50% shortening will still only take 5-10 my to evolve.

Stress Exercises

1. Given principal stresses of 45 Mpa and 80 Mpa in an area, with σ_1 horizontal and σ_2 vertical, what are the normal and shear stresses on a bedding surface dipping 30°?
2. The state of stress (two dimensional) is measured on two slots in a mine. One slot dips 32° east and has a normal compressive stress of 57MPa and a shear stress of 12 Mpa downdip. Another slot dips 84°E and has a normal compressive stress of 40 MPa and a shear stress of 3 Mpa downdip. What is the state of stress in this region of the mine? (ie., what are the principal stresses and their orientations)? Recall that the perpendicular bisector of a chord passes through the centre of the circle.
3. Given the principal stresses of magnitude 63 MPa and 94 MPa, what is the state of stress on a joint face whose normal is oriented at 53° to the σ_1 direction? The joint face will slide if the stress exceeds the static friction criterion:

   $\tau = 0.4\ \text{MPa} + \sigma_n \mu$ where μ, the coefficient of friction = 0.63

   Will the joint slide?

Answers to Stress exercise 1

1. Given principal stresses of 45 Mpa and 80 Mpa in an area, with σ_1 horizontal and σ_2 vertical, what are the normal and shear stresses on a bedding surface dipping 30°?

- σ_1 **= 80** Mpa; σ_2 **= 45** MPa
- plane makes an angle of **30**° with σ_1

Method 1 (formulae)

Substitute into formulae relating principal stresses to resolved shear and normal stresses on a plane

$$\sigma_n = ½ (\sigma_1 + \sigma_2) - ½ (\sigma_1 - \sigma_2)\cos 2\theta \qquad (1)$$

$$\tau = ½ (\sigma_1 - \sigma_2)\sin 2\theta \qquad (2)$$

i.e. σ_n = (80 + 45)/2 - cos 60 x (80-45)/2

= 125/2 -(0.5 x 35)/2 = 62.5-17.5/2 **= 53.75 MPa**

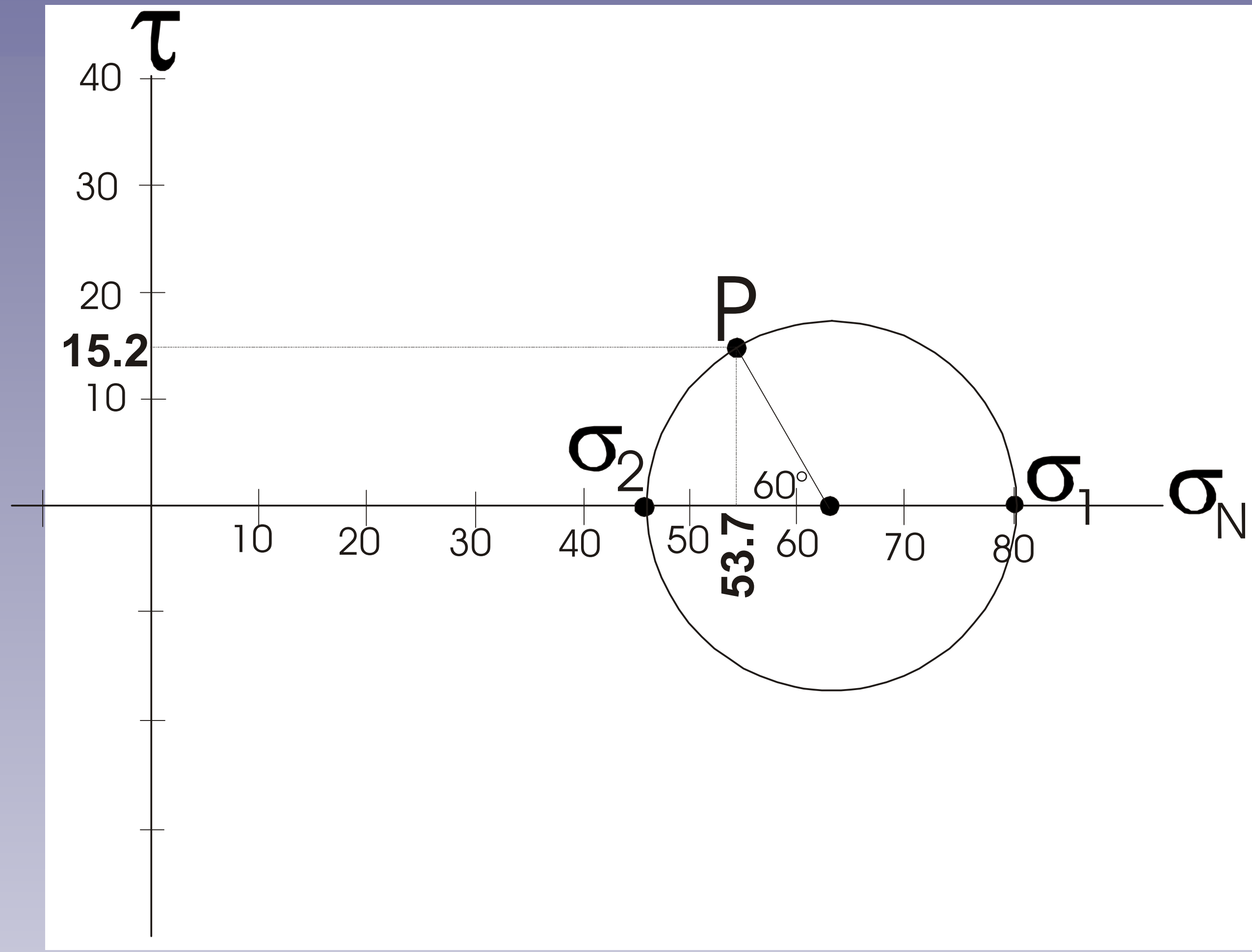
and

τ = sin 60 x (80-45)/2

= 0.866 x 35/2 **= 15.155 MPa**

Answers to Stress exercise 1 cont

- **Method 2** (graphical construction using Mohr circle)



Answers to Stress exercise 3

1. Given the principal stresses of magnitude 63 MPa and 94 MPa, what is the state of stress on a joint face whose normal is oriented at 53° to the σ_1 direction? The joint face will slide if the stress exceeds the static friction criterion:

   $\tau = 0.4$ MPa $+\sigma_n\mu$ where μ, the coefficient of friction = 0.63

   Will the joint slide?

Using either a Mohr Circle construction or the formulae it can be shown that the resolved normal and shear stresses on the joint are:

$\sigma_n = (94 + 63)/2 - \cos 74 \times (94-63)/2$

$= 157/2 -(0.276 \times 31)/2$ **= 74.22 MPa**

and

$\tau = \sin 74 \times (94-63)/2$

$= 0.96 \times 31/2$ **= 14.88 Mpa**

Now the criterion for sliding is that τ **> 0.4 +**$\mu\sigma_n$

i.e. τ **> 0.4 + 0.63 x 74.22 > 47.16 MPa**

But the resolved shear stress, τ, on this joint is only 14.88 MPa

- Therefore the joint face is stable and won't slide

Answers to Stress exercise 2

2. The state of stress (two dimensional) is measured on two slots in a mine. One slot dips 32° east and has a normal compressive stress of 57MPa and a shear stress of 12 Mpa down-dip. Another slot dips 84°E and has a normal compressive stress of 40 MPa and a shear stress of 3 Mpa down-dip. What is the state of stress in this region of the mine? (ie., what are the principal stresses and their orientations)? Recall that the perpendicular bisector of a chord passes through the centre of the circle.

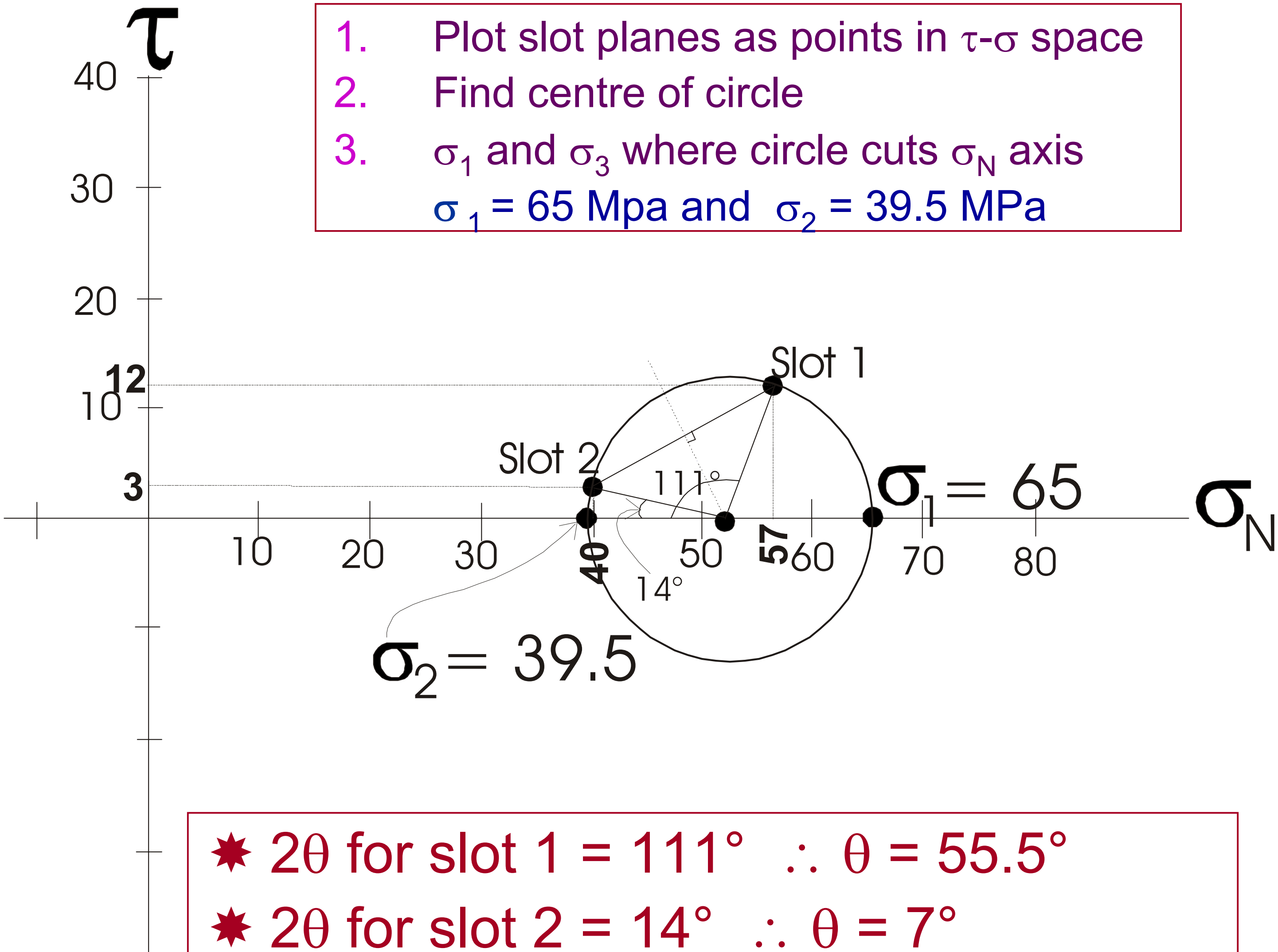
1. Plot the points representing the planes of Slots 1 and 2 on a Mohr diagram in τ-σ space using the measured shear and normal stresses
2. The centre of the Mohr circle (on the σ_N axis) can be found from the right bisector of the line between the points representing Slot 1 and Slot 2
3. Using this centre a circle can be constructed that passes through the points representing Slots 1 and 2

Thus…

Answers to Stress exercise 2 cont

1. Plot slot planes as points in τ-σ space
2. Find centre of circle
3. σ_1 and σ_3 where circle cuts σ_N axis

   σ_1 = 65 Mpa and σ_2 = 39.5 MPa



- 2θ for slot 1 = 111° ∴ θ = 55.5°
- 2θ for slot 2 = 14° ∴ θ = 7°

Answers to Stress exercise 2 cont

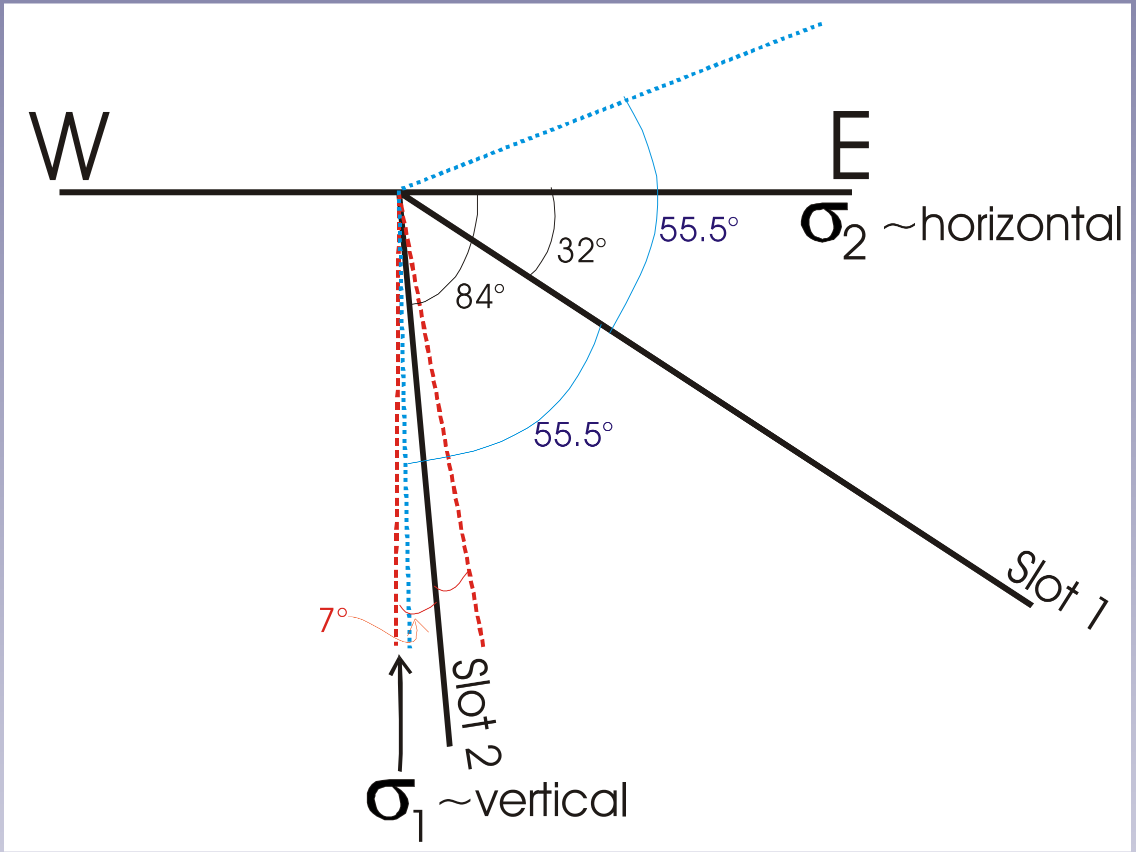
- The orientation of σ_1 and σ_2 can now be determined graphically:

2θ for slot 1 = 111° ∴ θ = 55.5°

2θ for slot 2 = 14° ∴ θ = 7°

- Note that there are two possible solution for the orientation of σ_1 for each slot but only one of these solutions coincides for **both** slots

- The solution that satisfies both is for σ_1 to be oriented approximately vertically and therefore σ_2 is approximately horizontal



Fracture questions

- Some of the fracture parameters for experimentally deformed Westerly Granite are:

τ_0 = 37 Mpa; T_0 = -21 Mpa; μ = 1.4

where τ_0 = cohesive (shear) strength; T_0 = Tensile strength; μ = Coulomb coefficient

1. Construct a Mohr diagram showing the Coulomb fracture criterion and tensile fracture strength of Westerly Granite
2. Plot the state of effective stress in Westerly Granite at a depth of 5 km, initially assuming that there are no differential stresses (i.e., the stress state is hydrostatic) and that the stress on the rock is due to the pressure of the overburden (i.e, the lithostatic pressure, ρgh, where ρ = 2620 kg/m^3 and g = 9.8 m/s^2).

   Consider , in turn, three different fluid pressures: (a) hydrostatic, where the fluid pressure is equal to the weight of the column of water extending from the surface (ρ_{H2O} = 1030 kg/m^3); (b) where the fluid pressure is 0.8 of the lithostatic pressure (i.e., λ = 0.8); and (c) where the fluid pressure equals the lithostatic pressure). [Note that because we are considering only hydrostatic stresses at this stage the states of stress will only be points on the Mohr diagram]

Fracture questions cont.

3. Now consider that the region above begins to undergo horizontal tectonic **compression**

   For each of the different scenarios of fluid pressure given above, plot a succession of Mohr circles illustrating the progressive states of stress up to the point at which the rock fails by fracture

At failure in each scenario:

a. What is the strength of the rock (differential stress at the point of failure) in each case?
b. In each case are the fractures that form shear fractures or extensional fractures?
c. If they are shear fractures then what is the predicted sense of displacement on each fracture?

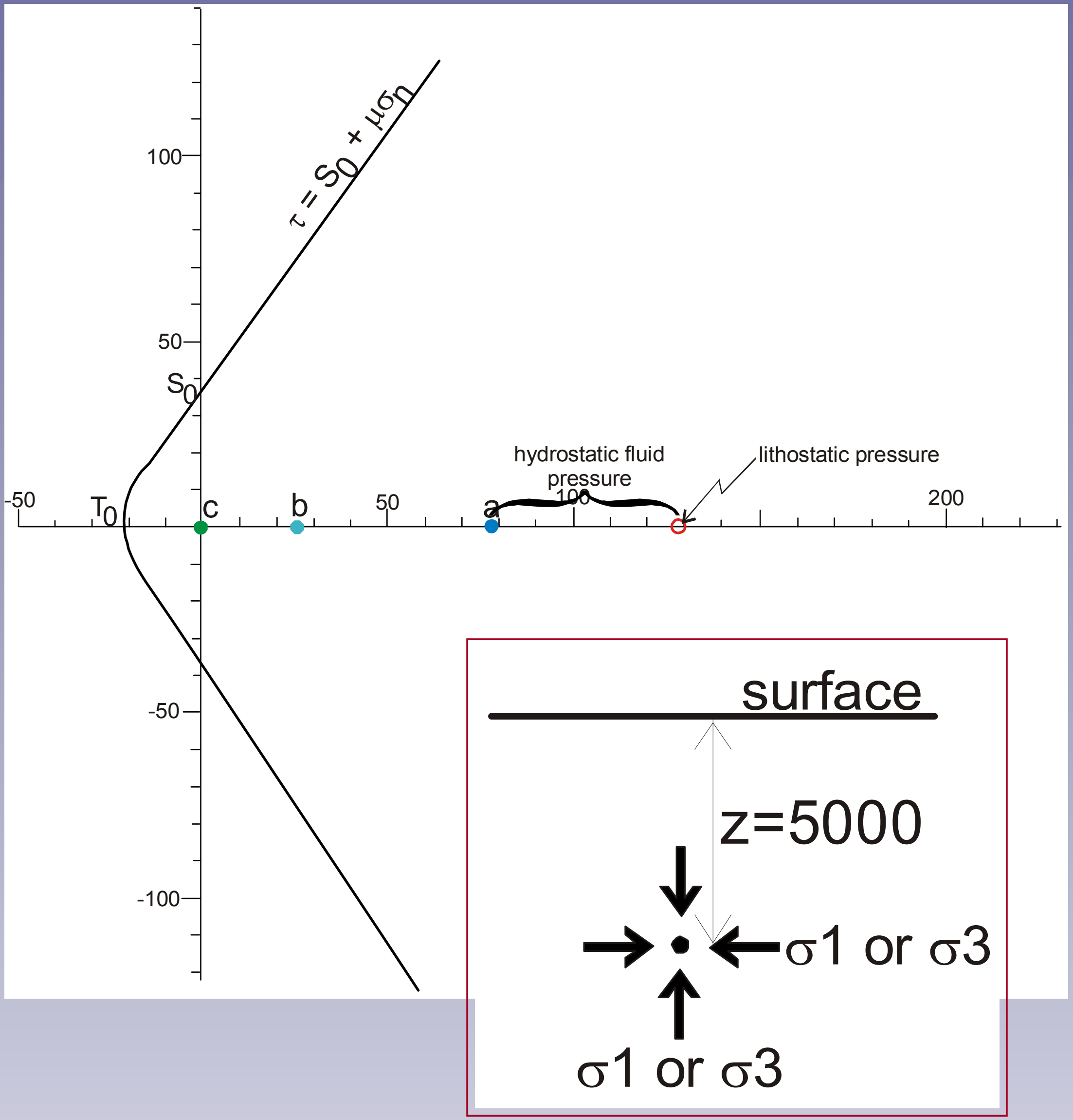
4. Now repeat the exercise for each scenario considering the region begins to undergo horizontal tectonic **extension**

Answers to Fractures exercise 1&2

lithostatic pressure =ρgh
= (2620 x 9.8 x 5000)/10^6 MPa
= 128.38 Mpa

hydrostatic fluid pressure
(= $\rho_f$gh
= (1030 x 9.8 x 5000)/10^6 MPa
= 50.47 Mpa

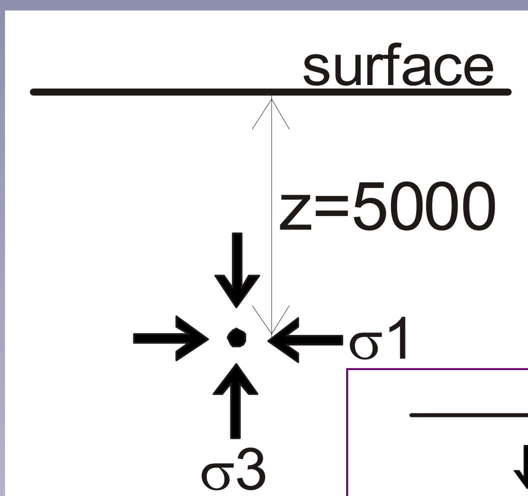
effective hydrostatic stress:
Case a: 128.38-50.47 MPa
= 77.91 MPa
Case b: 128.38 - (0.8 x 128.38)
= 25.68 MPa
Case c = 128.38 - 128.38 = 0 MPa

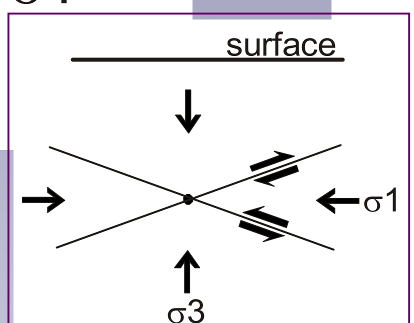


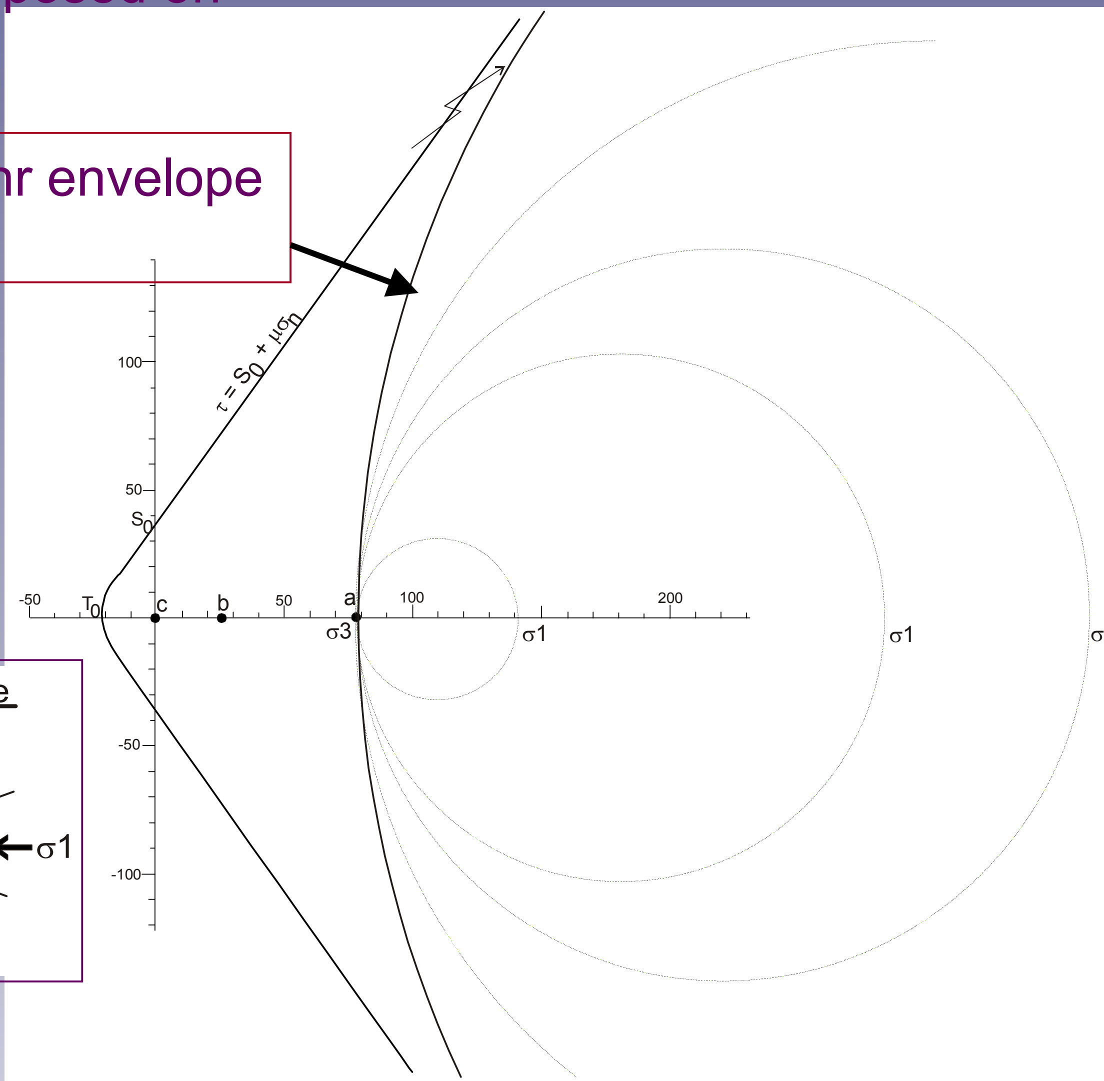
Answers to Fractures exercise 3 (i)

- Horizontal compression imposed on hydrostatic conditions at a

This circle is tangent to Mohr envelope
(σ_1 - σ_3 = ~700 MPa)





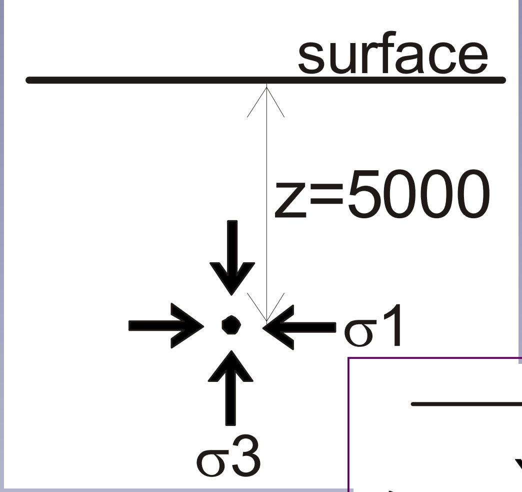


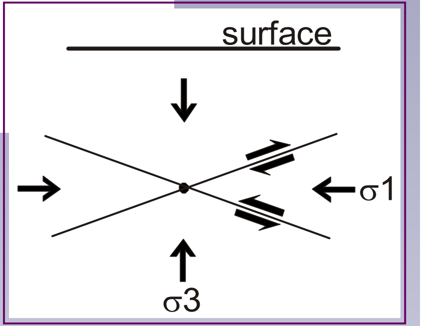
shear fractures (reverse sense) dipping ±17.5°

Answers to Fractures exercise 3 cont (ii)

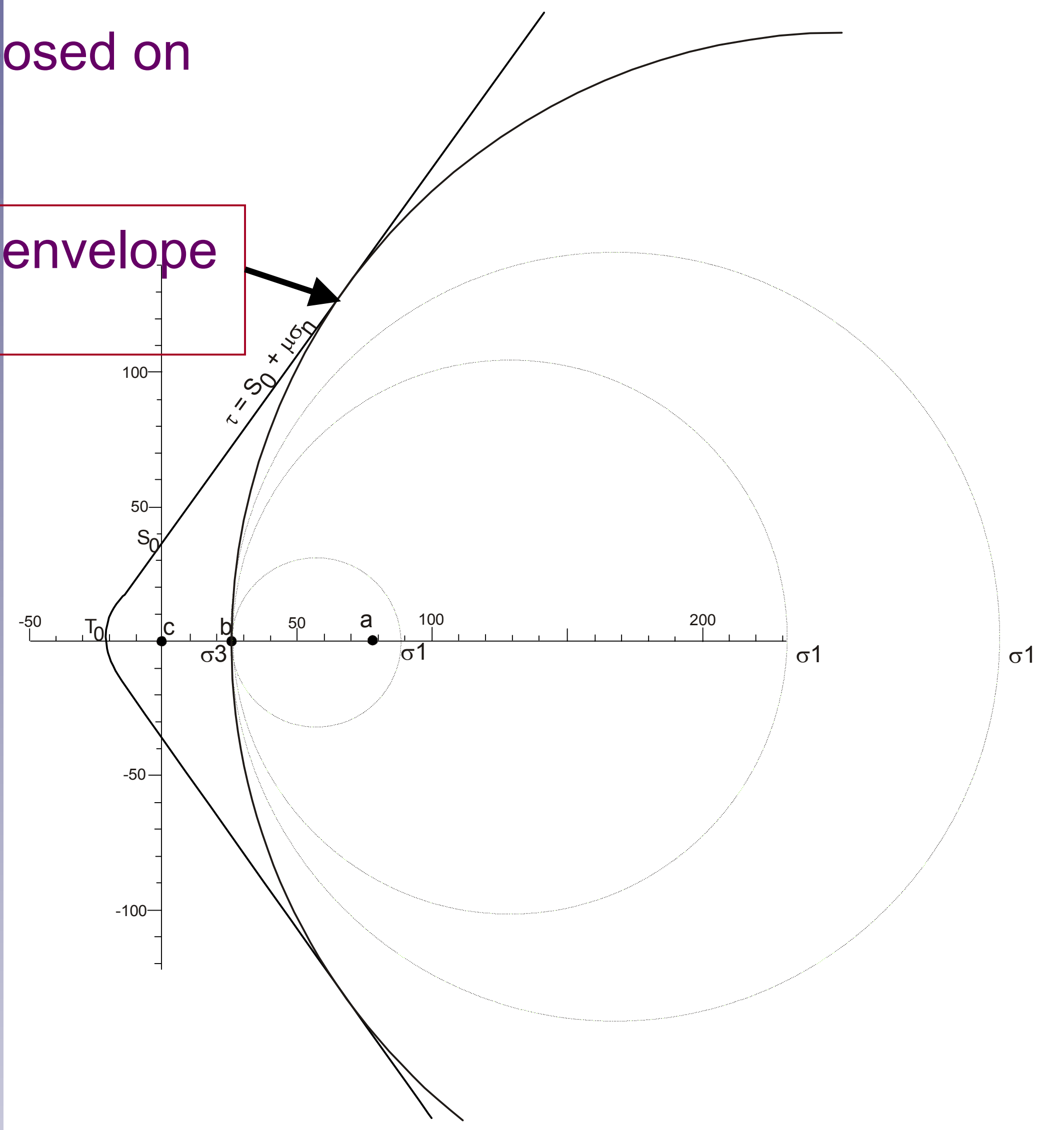
- Horizontal compression imposed on hydrostatic conditions at b

This circle is tangent to Mohr envelope ($\sigma_1 - \sigma_3$ = ~430 MPa)





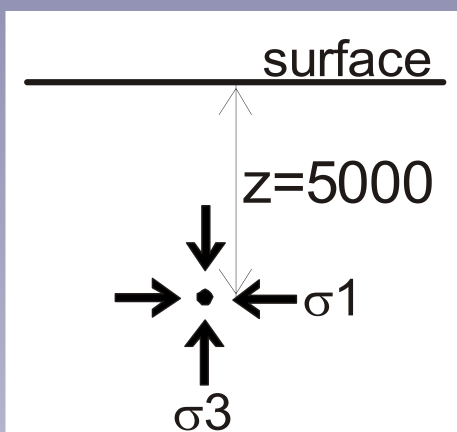
shear fractures (reverse sense) dipping ±17.5°

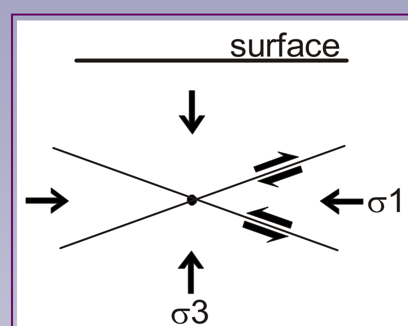


Answers to Fractures exercise 3 cont (iii)

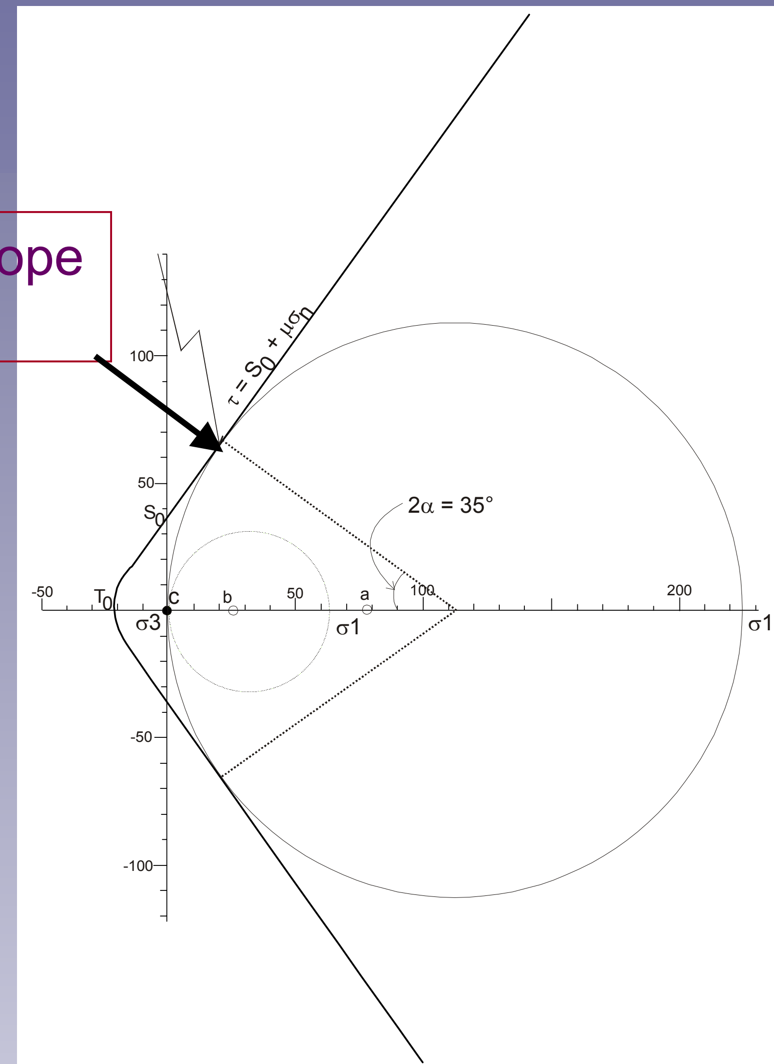
- Horizontal compression imposed on hydrostatic conditions at c

This circle is tangent to Mohr envelope ($\sigma_1 - \sigma_3$ = ~117 MPa)



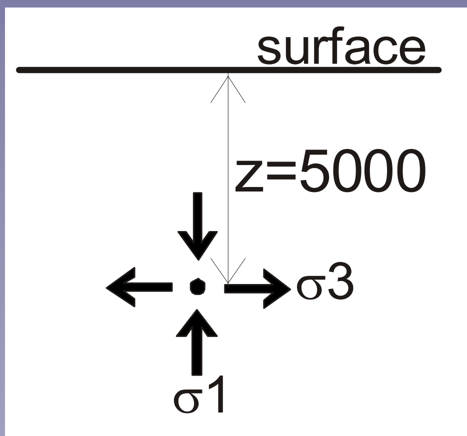


shear fractures (reverse sense) dipping ±17.5°

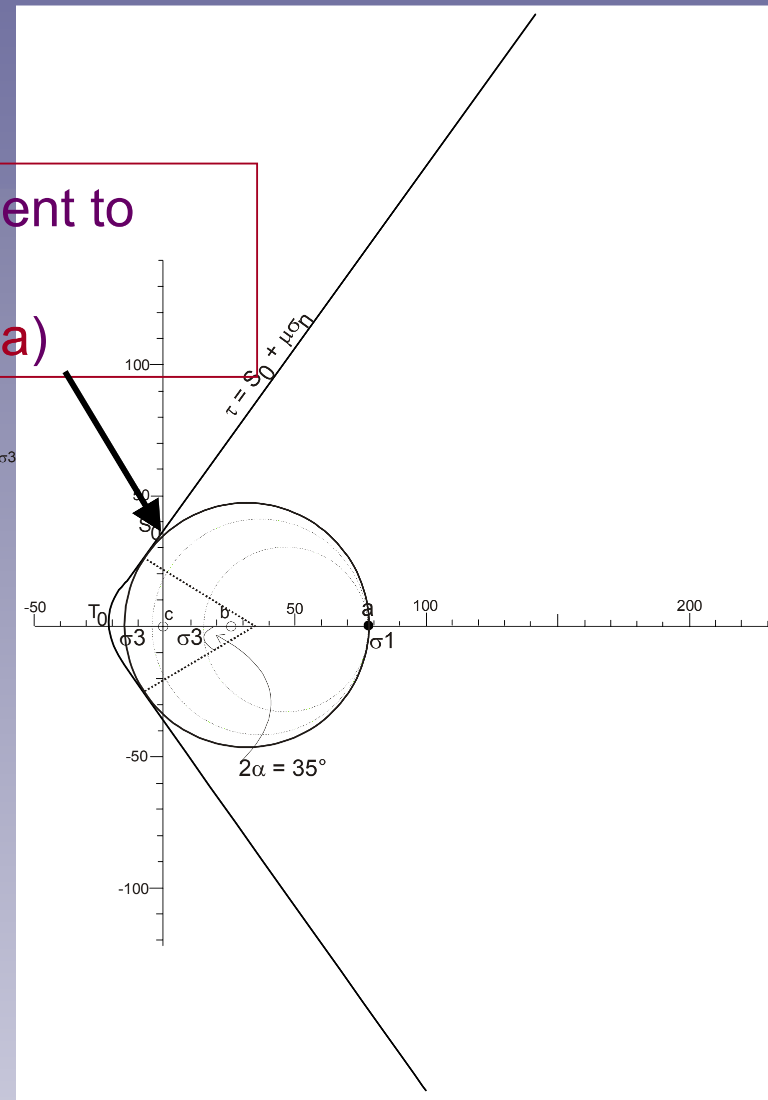


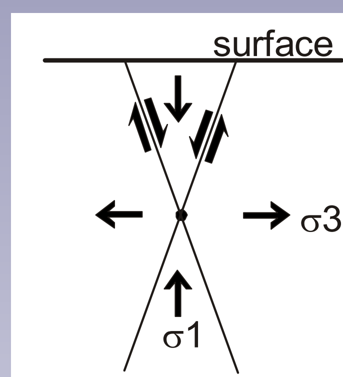
Answers to Fractures exercise 4 cont (iv)

- Horizontal tension imposed on hydrostatic conditions at a



This circle is tangent to Mohr envelope
($\sigma_1 - \sigma_3$ = ~94 MPa)

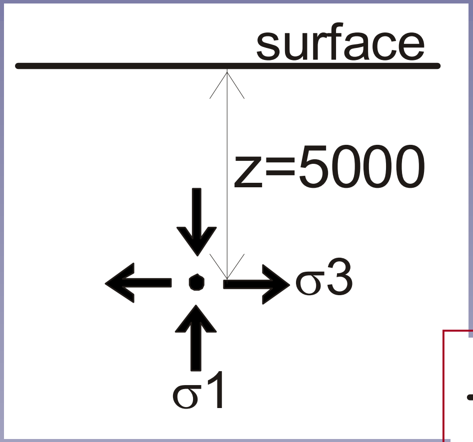




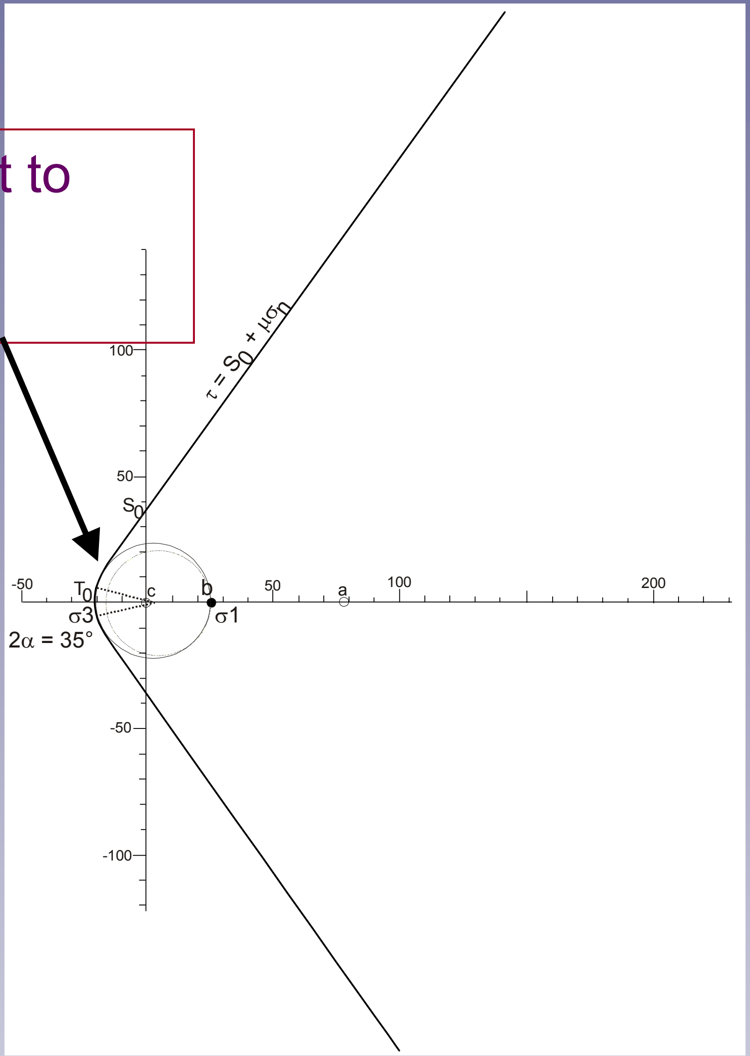
shear fractures (normal sense) dipping ±72.5°

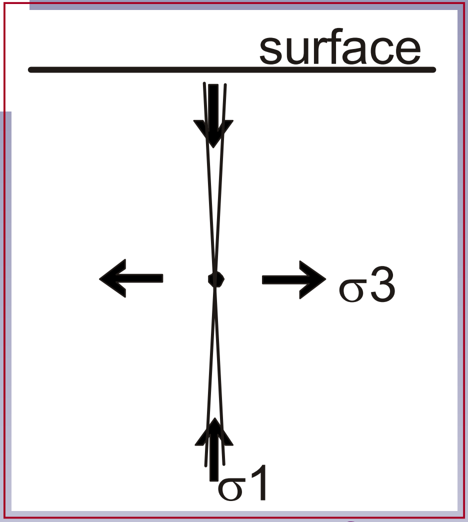
Answers to Fractures exercise 4 cont (v)

- Horizontal tension imposed on hydrostatic conditions at b



This circle is tangent to Mohr envelope ($\sigma_1 - \sigma_3$ = ~46 MPa)

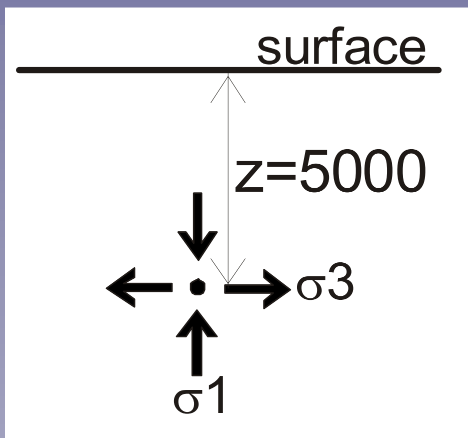




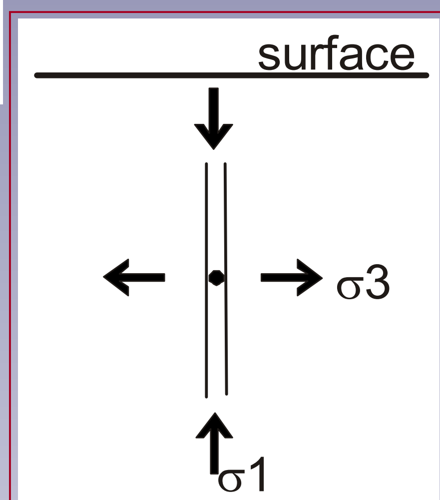
Extensional shear fractures (normal sense) dipping ±88°

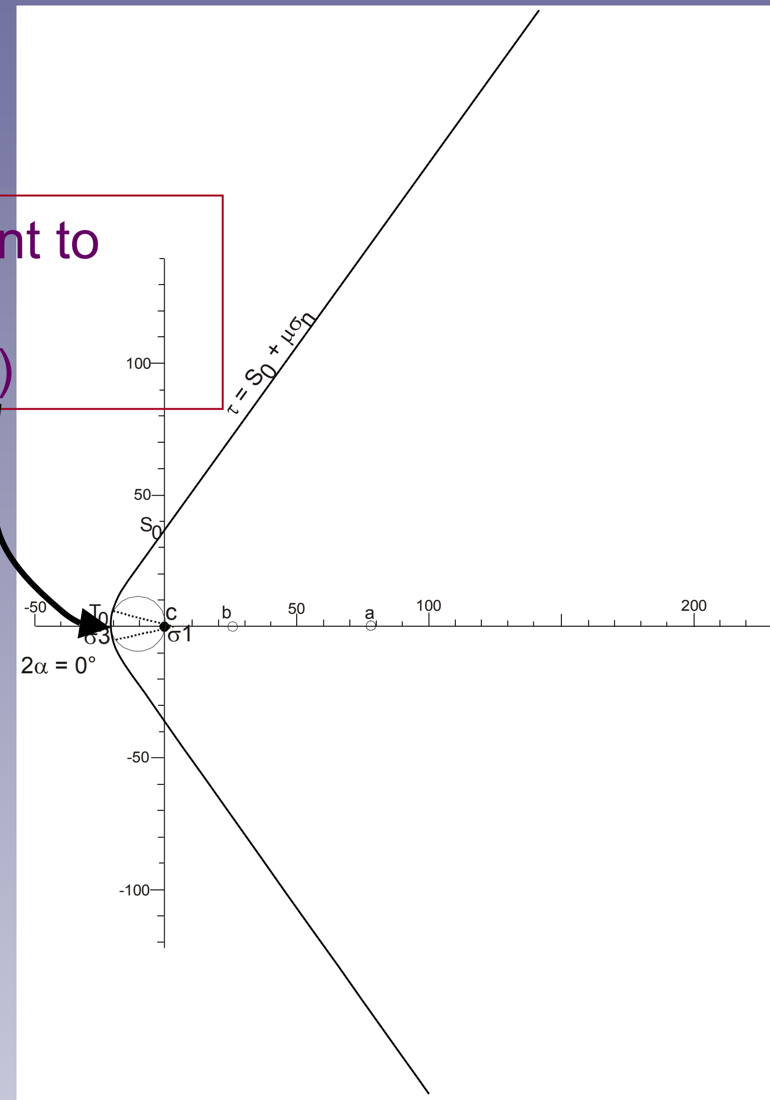
Answers to Fractures exercise 4 cont (vi)

- Horizontal tension imposed on hydrostatic conditions at c



This circle is tangent to Mohr envelope
($\sigma_1 - \sigma_3$ = ~21 MPa)





Pure extension fractures (normal sense) dipping 90° (vertical)